The Super-Sbox Cryptanalysis

Thomas Peyrin

CCRG seminar - Nanyang Technological University

Singapore - October 26, 2010

Outline

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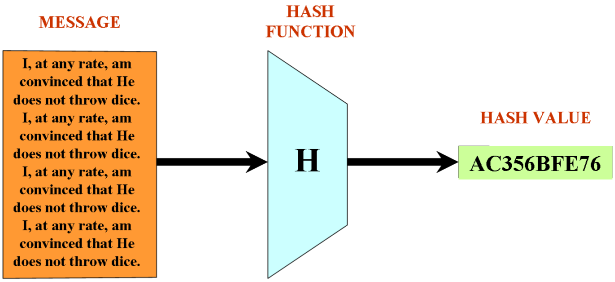
Introduction

The Super-Sbox attack

A case study: Grøstl (Gauravaram et al.)

Results and future works

What is a Hash Function ?



- H maps an **arbitrary length input** (the message M) to a **fixed length output** (typically $n = 128$, $n = 160$ or $n = 256$).
- no secret parameter.
- H must be easy to compute.

The security goals

- **pre-image resistance:** given an output challenge y, the attacker can not find a message x such that $H(x) = y$, in less than $\theta(2^n)$ operations.
- **2nd pre-image resistance:** given a challenge (x, y) so that $H(x) = y$, the attacker can not find a message $x' \neq x$ such that $H(x') = y$, in less than $\theta(2^n)$ operations.
- **collision resistance:** the attacker can not find two messages (x, x') such that $H(x) = H(x')$, in less than $\theta(2^{n/2})$ operations (a generic attack with the birthday paradox exists [Yuval-79]).

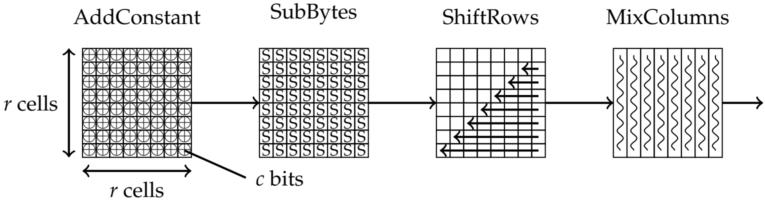
SHA-3 competition

The SHA-3 hash function competition:

- started in October 2008, 64 submissions
- 51 candidates accepted for the first round
- 14 semi-finalists selected in 2009
- 4/5/6 finalists to be selected end 2010
- winner to be announced in 2012

Among the 14 semi-finalists, one can identify 4 AES-based candidates. For example ECHO and Grøstl.

What is an AES-like permutation ?



$$MixColumns \circ ShiftRows \circ SubBytes \circ AddConstant(C)$$

- **AddConstant:** in known-key model, just add a round-dependent constant (breaks natural symmetry of the three other functions)
- **SubBytes:** application of a c-bit Sbox (only non-linear part)
- **ShiftRows:** rotate column position of all cells in a row, according to its row position
- **MixColumns:** linear diffusion layer.

Hash function collision attacks

In general, there are **two basic tools** in order to find a collision: the differential path building technique and the freedom degree utilization method.

The differential path building techniques (for `SHA-1`):

- local collisions
- linear perturbation mask
- non-linear parts

The freedom degree utilization methods (for `SHA-1`):

- neutral bits
- message modifications
- boomerang trails

Hash function collision attacks

In general, there are **two basic tools** in order to find a collision: the differential path building technique and the freedom degree utilization method.

The differential path building techniques (for AES-based):

- truncated differential paths

The freedom degree utilization methods (for AES-based):

- rebound attacks
- multiple-inbound attacks
- start-from-the-middle attacks
- super-Sbox attacks

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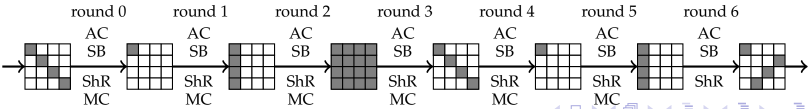
Truncated differences

- Originally introduced by Knudsen for block ciphers [Knudsen FSE 1994]
- Later applied to hash functions (collision attack on Grindahl) [Peyrin ASIACRYPT 2007]
- **Idea:** consider byte-differences, without considering their actual value (active or inactive).
- **Only the truncated differences propagation through MixColumns behave probabilistically. Per column:**

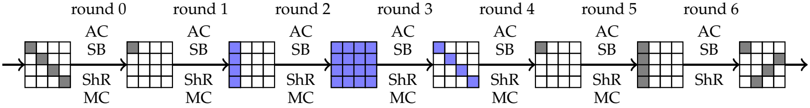
$$\text{nb active input cells} + \text{nb active output cells} \geq r + 1.$$

$$P \simeq 2^{-xc} \text{ for } x \neq r \text{ inactive output cells.}$$



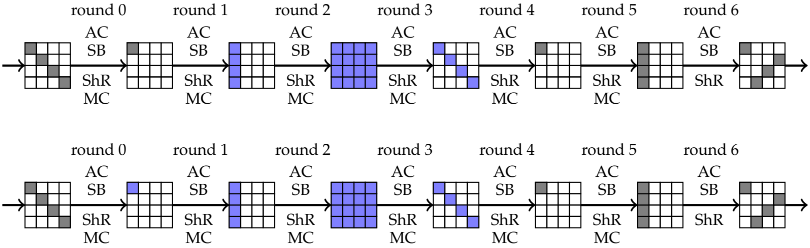
Controlled and uncontrolled rounds

- **Idea:** use the freedom degrees in the middle of the differential path).
- The path is divided into two different kind of steps:
  - **The controlled rounds:** the part where the freedom degrees are used (usually in the middle of the path). On average, finding a solution for the controlled rounds should cost only a few operations.
  - **The uncontrolled rounds:** the part where all the events are verified probabilistically (left and right part of the path) because no more freedom degree is available. Determine the complexity of the overall attack.



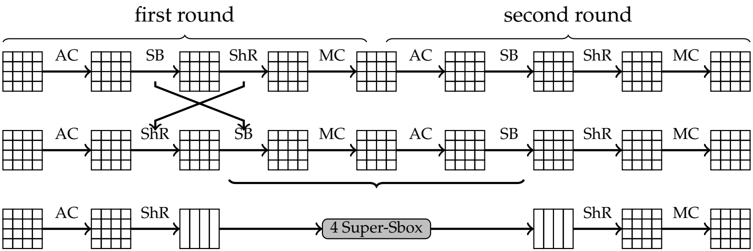
Rebound Attack and Start-from-the-middle

- **Rebound attack:** allows to get 2 controlled rounds [Mendel et al. FSE 2009]. Requires 2^{rc} memory. It broke compression functions of many SHA-3 candidates.
- **Start-from-the-middle:** use more complicated techniques to get up to 3 controlled rounds in the case of low weight differential paths [Mendel et al. SAC 2009]. Requires 2^{rc} memory.



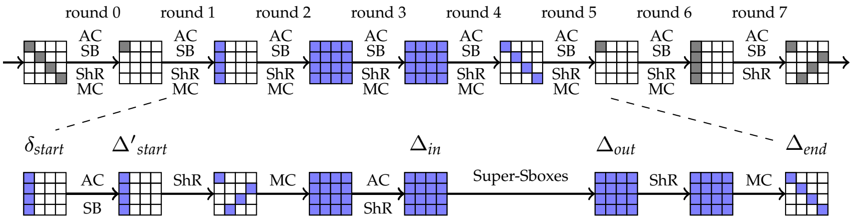
The Super-Sbox view

- Introduced by Daemen and Rijmen (e.g. [Daemen Rijmen SCN 2006]) to simplify the analysis of AES differential properties and not for cryptanalysis purposes.
- **Idea:** one can view two rounds of an AES-like permutation as a layer of big 2^{rc}-bit Sboxes preceded and followed by simple affine transformations. We call those **Super-Sboxes**



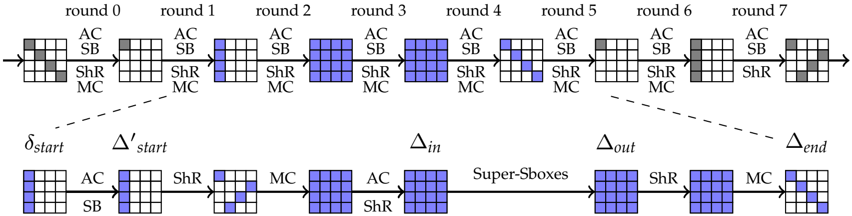
The controlled rounds in the Super-Sbox view

- **One can get 3 controlled rounds, even for high weight differential paths.**
- **Forward:** start with a random (not truncated) difference δ'_{start} at the beginning of round 2 (such that we obtain a compatible truncated difference Δ_{start} when inverting *SB* and *AC*). Then, pass *ShR*, *MC*, *AC* and *ShR* to obtain the aimed input difference Δ_{in} on the r Super-Sboxes.
- **Backward:** start with a random (not truncated) difference Δ_{end} at the end of round 4, and invert *MC* and *ShR* in order to obtain the aimed output difference Δ_{out} on the r Super-Sboxes.
- **Problem:** need the ability to find for each of the r columns, a value that maps Δ_{in} to Δ_{out} ... seems hard.



The controlled rounds

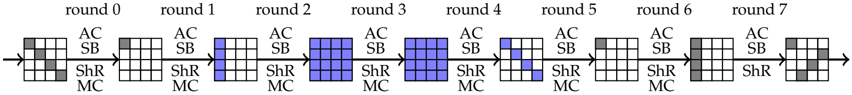
- **Idea:** pay a big price (2^{rc} operations and memory), but get many solutions (2^{rc}) once you paid.
- **1st step:** Fix a random Δ'_{start} difference value, which gives a fixed random Δ_{in}. For each of the r Super-Sboxes, exhaust all 2^{rc} possible actual values, then sort the results in r tables according to the output difference obtained.
- **2nd step:** try 2^{rc} distinct Δ_{end} differences. Then, for each Δ_{out} obtained by computing backward, check if for all the r columns the appropriate 2^{rc}-bit difference is present in the corresponding table. On average, one solution is found per Δ_{end} try.
- **The average complexity for finding one internal state pair verifying the controlled rounds is** 1**.**



The uncontrolled rounds

8-round path:

- On the left side, one has one $4 \mapsto 1$ MixColumns transition to control (round 1): $P \simeq 2^{-(r-1)c}$
- On the right side, one has one $4 \mapsto 1$ MixColumns transition to control (round 5): $P \simeq 2^{-(r-1)c}$
- Total complexity for finding a solution for the whole path: $2^{2(r-1)c}$ operations.



One has also to check that we have enough freedom degrees, such that a valid pair can be found.

Limited-birthday distinguishers

What is the generic complexity for mapping i fixed-difference bits to j fixed-difference bits through a random permutation E ?

Wlog, assume that $i \geq j$ and let $n := r^2c$. Due to the birthday paradox, each structure of 2^{n-i} input values obtained by fixing the value of the i fixed-difference bits allows to get fixed-difference on $2(n-i)$ output bits:

- if $j \leq 2(n-i)$, then one can select $2^{j/2}$ input values from one single structure and this suffices to achieve a collision on the j target positions. The attack complexity is about $2^{j/2}$.
- if $j > 2(n-i)$, then about $2^{j-2(n-i)}$ structures have to be used to obtain a collision on the j prescribed positions. Overall, the complexity of the attack is about $2^{n-i} \times 2^{j-2(n-i)} = 2^{i+j-n}$.

Same reasoning for the $n-j$ free difference bits on the output and attacking E^{-1}:

- if $i \leq 2(n-j)$, then the attack complexity is about $2^{i/2}$.
- if $i > 2(n-j)$, then the attack complexity is about 2^{i+j-n}.

Final complexity: $\max\{2^{j/2}, 2^{i+j-n}\}$.

Results on AES and Grøstl

Table: Results on the underlying permutation

target	rounds	computational complexity	memory requirements	type	source
AES	7	2^{24}	2^{16}	known-key-dist.	[Mendel et al. SAC 2009]
	8	2^{48}	2^{32}	known-key-dist.	**[Gilbert Peyrin FSE 2010]**
Grøstl-256 permutation	7	2^{56}		distinguisher	[Mendel et al. SAC 2009]
	8	2^{112}	2^{64}	distinguisher	**[Gilbert Peyrin FSE 2010]**

Table: Results on the compression function

target	rounds	computational complexity	memory requirements	type	source
Grøstl-256 compression function	6	2^{120}	2^{64}	semi-free-start coll.	[Mendel et al. FSE 2009]
	6	2^{64}	2^{64}	semi-free-start coll.	[Mendel et al. SAC 2009]
	7	2^{120}	2^{64}	semi-free-start coll.	**[Gilbert Peyrin FSE 2010]**
	7	2^{56}		distinguisher	[Mendel et al. SAC 2009]
	8	2^{112}	2^{64}	distinguisher	**[Gilbert Peyrin FSE 2010]**

* Results also independently obtained by Lamberger et al.

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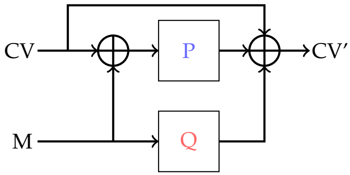
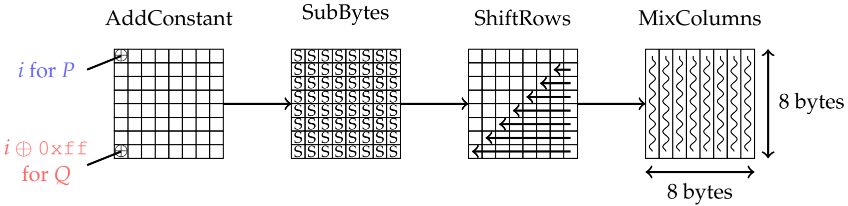
Grøstl compression function



Round i of permutations P and Q:

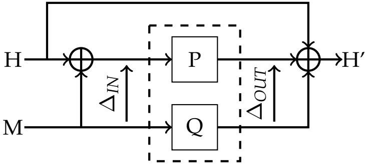


$$MixColumns \circ ShiftRows \circ SubBytes \circ AddConstant(C)$$

The internal differential attack

Problem: all previous attacks build classical differential paths for the permutation P and Q (allows to reach 8/10 rounds)

Idea: **look at the difference between the two parallel branches**
It works well on `Grøstl` because P and Q are almost identical (only the constant addition differs)



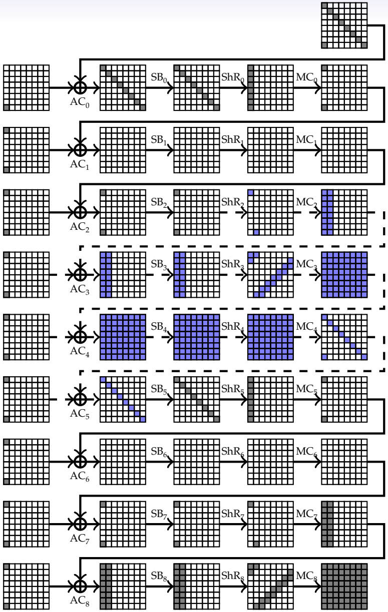
attacked primitive

Let A and B be s.t. $A \oplus B = \Delta_{IN}$ and $Q(A) \oplus P(B) = \Delta_{OUT}$

We have $h(H, M) = \Delta_{IN} \oplus \Delta_{OUT}$

What can we do with such a pair A and B ?

- **Distinguishing attack:**
  - assume Δ_{IN} is maintained in a set of x elements
  - assume Δ_{OUT} is maintained in a set of y elements
  - thus $h(H, M)$ is maintained in a set of $k = x \cdot y$ elements
  - we can distinguish the `Grøstl` compression function from an ideal one: such pair (H, M) can be generically obtained with $2^n/k$ computations
  - one can also distinguish the permutations P and Q from ideal permutations (with "limited birthday distinguishers")
- **Collision attack:**
  - because of a lack of freedom degrees, no improvement for the compression function attacks
  - but we can attack 5/10 rounds of the hash function



An example with 9 rounds:

- we have
  - $x = 2^{56}$
  - $y = 2^{128}$
  - $k = 2^{184}$
- thus the generic complexity is $2^{512-184} = 2^{328}$ operations
- we can find a valid candidate with only 2^{80} computations and 2^{64} memory
- the amount of freedom degrees only allows us to compute one such candidate, but generalization of the internal differential attack gives additional freedom degrees

Results for Grøstl

target	rounds	computational complexity	memory requirements	type	section
Grøstl-256 comp. function	7/10	2^{56}		distinguisher	[Mendel et al. SAC 2009]
	8/10	2^{112}	2^{64}	distinguisher	[Gilbert Peyrin FSE 2010]
	9/10	2^{80}	2^{64}	distinguisher*	**[Peyrin CRYPTO 2010]**
	10/10	2^{192}	2^{64}	distinguisher*	**[Peyrin CRYPTO 2010]**
Grøstl-512 comp. function	11/14	2^{640}	2^{64}	distinguisher*	**[Peyrin CRYPTO 2010]**
Grøstl-256 hash function	4/10	2^{64}	2^{64}	collision	[Mendel et al. SAC 2010]
	5/10	2^{79}	2^{64}	collision	**[Peyrin CRYPTO 2010]**
Grøstl-512 hash function	5/14	2^{176}	2^{64}	collision	[Mendel et al. SAC 2010]
	6/14	2^{177}	2^{64}	collision	**[Peyrin CRYPTO 2010]**

* for these distinguishers, the amount of available freedom degrees allows us to generate only one valid candidate with good probability

Be careful when designing a scheme:

also check the differential paths between the internal branches

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Results and future works

The Super-Sbox method:

- a very easy-to-use yet powerful cryptanalysis tool
- provides the best attack against 128-bit AES in the known key model
- also very efficient against AES-based hash functions: ECHO, Grøstl, ... In particular, first distinguishing attack against full Grøstl-256 compression function or internal permutations

Future works:

- find better differential paths for ECHO ([Sasaki et al. - ASIACRYPT 2010] [Schläffer - SAC 2010])
- derive collision attacks for the Grøstl hash function with internal differential paths ([Ideguchi et al. - eprint 2010])
- try to apply Super-Sbox attack to other schemes (work on SHAvite-3 to be published soon)
- switching attack: switch completely the type of differential path considered between the left and the right controlled rounds and use the Super-Sbox setting in order to link them